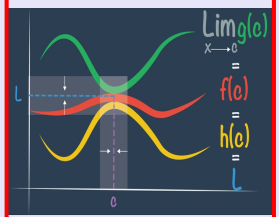
Calculus I

Lecture 23



Feb 19-8:47 AM

Find $f'(x)$ for $f(x) = x^3 \sin x + 5$

$$f'(x) = \frac{d}{dx}[f(x)]$$

$$= \frac{d}{dx}[x^3 \sin x + 5]$$

$$= \frac{d}{dx}[\underbrace{x^3 \sin x}_{\text{Product Rule}}] + \frac{d}{dx}[5]$$

$$= \underbrace{\frac{d}{dx}[x^3]}_{\text{Power Rule}} \cdot \sin x + x^3 \cdot \frac{d}{dx}[\sin x] + 0$$

$$= \boxed{3x^2 \sin x + x^3 \cos x}$$

Oct 8-7:32 AM

Find $f''(x)$ for $f(x) = \frac{x+5}{x-5}$

f-Double Prime ⇒ Second derivative

Derivative of first derivative

$$f''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$$

$$f'(x) = \frac{\frac{d}{dx}[x+5]\cdot(x-5) - (x+5)\cdot\frac{d}{dx}[x-5]}{(x-5)^2}$$

$$f'(x) = \frac{1(x-5)-(x+5)\cdot 1}{(x-5)^2} = \frac{-10}{(x-5)^2}$$

$$f''(x) = \frac{d}{dx}\left[f'(x)\right] = \frac{d}{dx}\left[\frac{-10}{(x-5)^2}\right]$$

$$= \frac{\cancelto{0}{\frac{d}{dx}[-10]}\cdot(x-5)^2 - (-10)\cdot\frac{d}{dx}\left[(x-5)^2\right]}{\left[(x-5)^2\right]^2}$$

do chain Rule

$$= \frac{10\,\frac{d}{dx}\left[(x-5)^2\right]}{(x-5)^4}$$

$$\frac{d}{dx}\left[(x-5)^2\right] =$$

$$\frac{d}{dx}\left[(x-5)(x-5)\right] =$$

$$1\cdot(x-5)+(x-5)\cdot 1 =$$

$$2(x-5)$$

$$= \frac{10\cdot 2\cancel{(x-5)}}{(x-5)^{\cancel{4}\,3}} = \boxed{\frac{20}{(x-5)^3}}\ f''(x)$$

Oct 8-7:36 AM

$$\frac{d}{dx}\left[\sec x\right] = \frac{d}{dx}\left[\frac{1}{\cos x}\right]$$

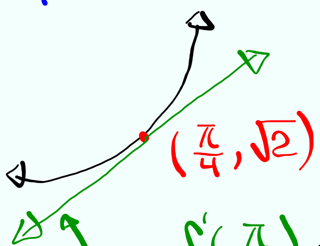
$$= \frac{\cancelto{0}{\frac{d}{dx}[1]}\cdot\cos x - 1\cdot\cancelto{-\sin x}{\frac{d}{dx}[\cos x]}}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x} = \tan x\cdot\sec x$$

$$\boxed{\frac{d}{dx}\left[\sec x\right] = \sec x\tan x}$$

Oct 8-7:47 AM

Find the $\boxed{\text{eqn of the tan. line}}$ to the graph of $f(x) = \sec x$ at $x = \frac{\pi}{4}$.



$f\left(\frac{\pi}{4}\right) = \sec\frac{\pi}{4} = \sqrt{2}$

$f'(x) = \sec x \tan x$

$m = f'\left(\frac{\pi}{4}\right) = \sec\frac{\pi}{4} \tan\frac{\pi}{4}$

$= \sqrt{2} \cdot 1 = \sqrt{2}$

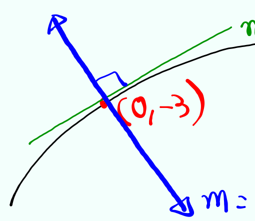
$y - y_1 = m(x - x_1)$

$y - \sqrt{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right)$

$\boxed{y = \sqrt{2}x - \frac{\pi\sqrt{2}}{4} + \sqrt{2}}$

Oct 8-7:50 AM

Find eqn of the normal line to the graph of $f(x) = \frac{2x+3}{x-1}$ at the $\underbrace{\text{y-int.}}_{x=0}$



$m = f'(0)$

$= \frac{-5}{(0-1)^2} = -5$

$f(0) = -3$

Tan. Point $(0,-3)$

$m = \frac{-1}{m_{\text{tan. line}}} = \frac{-1}{-5} = \boxed{\frac{1}{5}}$

$f'(x) = \frac{2(x-1) - (2x+3)\cdot 1}{(x-1)^2} = \frac{2x-2-2x-3}{(x-1)^2} = \frac{-5}{(x-1)^2}$

$y - y_1 = m(x - x_1)$

$y - (-3) = \frac{1}{5}(x - 0)$

$\boxed{y = \frac{1}{5}x - 3}$

Normal line at $(0,-3)$

Oct 8-7:57 AM

Find $\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right]$

$$= \frac{\underbrace{\frac{d}{dx}[\cos x]}_{-\sin x}\cdot \sin x - \cos x \cdot \underbrace{\frac{d}{dx}[\sin x]}_{\cos x}}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-1(\underbrace{\sin^2 x + \cos^2 x}_{1})}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

$$\boxed{\begin{aligned} \frac{d}{dx}[\cot x] &= -\csc^2 x \\ \frac{d}{dx}[\cos x] &= -\sin x \\ \frac{d}{dx}[\csc x] &= -\csc x \cot x \end{aligned}}$$

Oct 8-8:04 AM

Prove $\frac{d}{dx}[c f(x)] = c\frac{d}{dx}[f(x)]$

Let $g(x) = c f(x)$

$$g'(x) = \lim_{h\to 0}\frac{g(x+h) - g(x)}{h} = \lim_{h\to 0}\frac{c f(x+h) - c f(x)}{h}$$

$$= \lim_{h\to 0}\frac{c\left(f(x+h) - f(x)\right)}{h}$$

$$= c\left\{\lim_{h\to 0}\frac{f(x+h) - f(x)}{h}\right\}$$

$$= c\,\frac{d}{dx}[f(x)]$$

Oct 8-8:11 AM

Prove $\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$

Let $H(x) = f(x) + g(x)$

$$H'(x) = \lim_{h \to 0} \frac{H(x+h) - H(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right]$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

Oct 8-8:15 AM

Prove $\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]$

$H(x) = f(x) \cdot g(x)$

$$H'(x) = \lim_{h \to 0} \frac{H(x+h) - H(x)}{h} = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)\,g(x+h) \overbrace{- f(x)g(x+h) + f(x)g(x+h)}^{0} - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{g(x+h)[f(x+h) - f(x)]}{h} + \frac{f(x)[g(x+h) - g(x)]}{h} \right]$$

$$= \lim_{h \to 0} \underbrace{g(x+h)}_{\to g(x)} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} f(x) \frac{g(x+h) - g(x)}{h}$$

$$= g(x) \cdot f'(x) + f(x) \cdot g'(x)$$

$$= f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\boxed{\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]}$$

Oct 8-8:21 AM

Google or look up for proof

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{[g(x)]^2}$$

Quotient Rule

Oct 8-8:32 AM